

Accuracy/Computation Performance of a New Trilateration Scheme for GPS-Style Localization

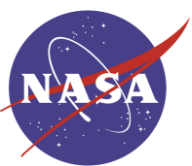
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March 04-10, 2018

**2013 IEEE Aerospace Conference
Big Sky, Montana**



Outline of Talk

INTRODUCTION

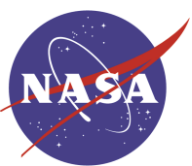
THEORY AND ANALYSIS

- **NEWTON-RAPHSON SCHEME AND GEOMETRIC TRILATERATION SCHEME**
- **MAJOR ERROR SOURCES**

SIMULATIONS OF PERFORMANCE

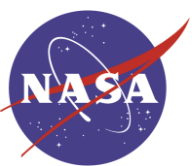
- **A NOTIONAL MARS NAVIGATION SATELLITE SYSTEM**
- **ACCURACY/COMPUTATION PERFORMANCE COMPARISON AND OBSERVATIONS**

CONCLUSION AND FUTURE WORK



Introduction

- The United States' Global Positioning System (GPS) has an estimated development and deployment cost of \$33 billion and the annual operation cost of \$1 billion
- Most localization schemes are based on trilateration – estimation of position based on range measurements
- Most commonly used trilateration scheme is Newton-Raphson (NR) Method
- This paper describes a new trilateration scheme that differs from NR scheme:
 - The new Geometric Trilateration (GT) scheme is derived from Pythagoras Theorem, whereas the NR scheme is based on the principle of linear regression
 - The NR method uses the absolute locations (x_n, y_n, z_n) 's of the GPS satellites as input to each step of the localization computation. The GT method uses the Directional Cosines U_i 's from Earth's center to the GPS satellite S_i
 - Both the NR and the GT method iterate to coverage to a solution. In each iteration step, multiple matrix operations are performed. The NR method constructs a different matrix in each iteration step. The GT scheme uses the same matrix in each iteration, thus requiring computing the matrix only once for all subsequent iterations



Theory/Analysis: Newton-Raphson Method

$$\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} + cDt = d_i$$

Given sat_i and ρ_i for $i = 1, \dots, n$; $n \geq 4$

Initialize $\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $T = 0$

LS Iterative Scheme:

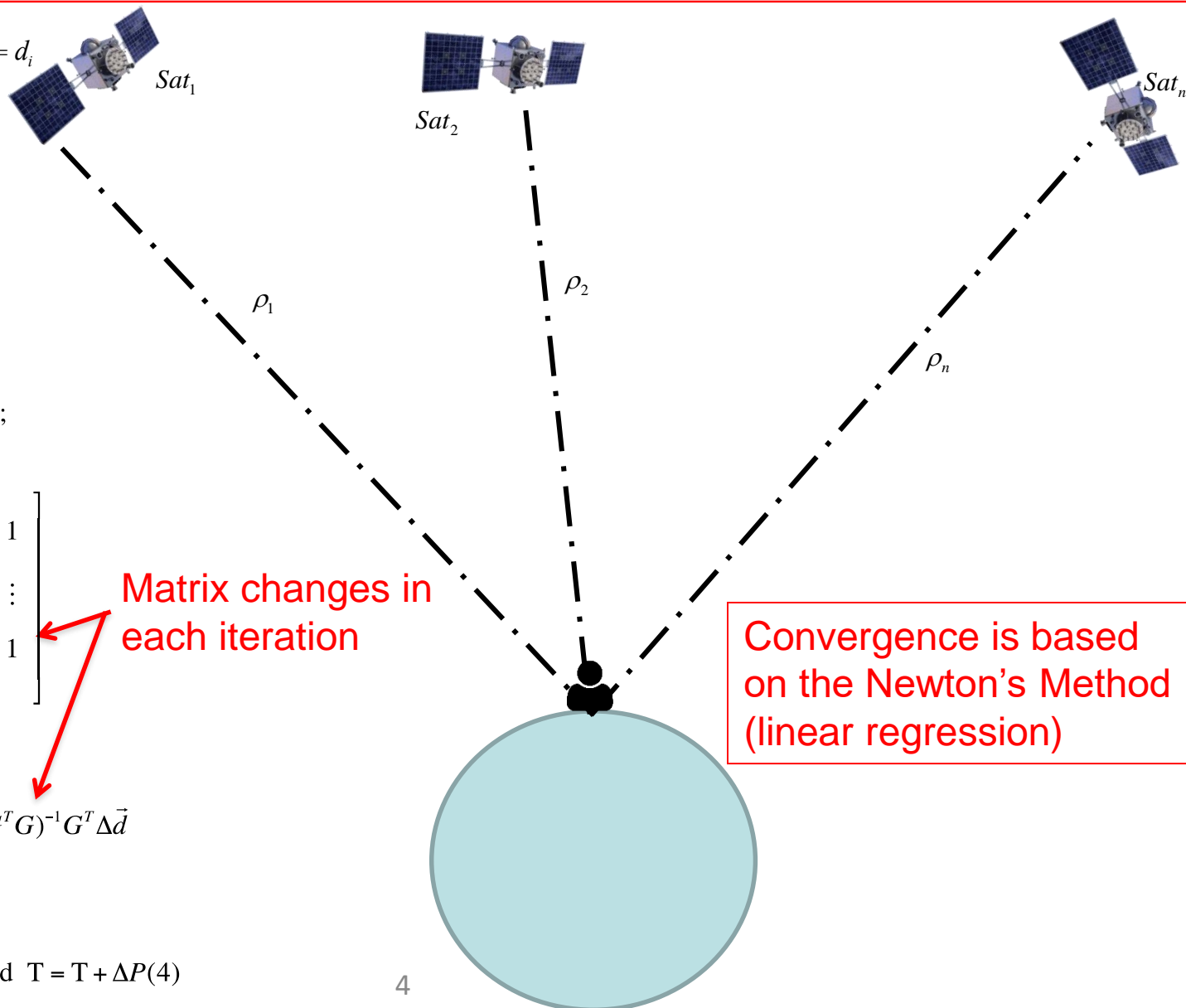
$$r_i = \left\| sat_i - \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} \right\| \text{ for } i = 1, \dots, n;$$

$$G = \begin{bmatrix} \frac{x_1 - \tilde{x}}{r_1} & \frac{y_1 - \tilde{y}}{r_1} & \frac{z_1 - \tilde{z}}{r_1} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_n - \tilde{x}}{r_n} & \frac{y_n - \tilde{y}}{r_n} & \frac{z_n - \tilde{z}}{r_n} & 1 \end{bmatrix}$$

Matrix changes in each iteration

$$\Delta \vec{d} = \begin{bmatrix} \rho_1 - r_1 + T \\ \vdots \\ \rho_n - r_n + T \end{bmatrix}; \quad \Delta \vec{P} = (G^T G)^{-1} G^T \Delta \vec{d}$$

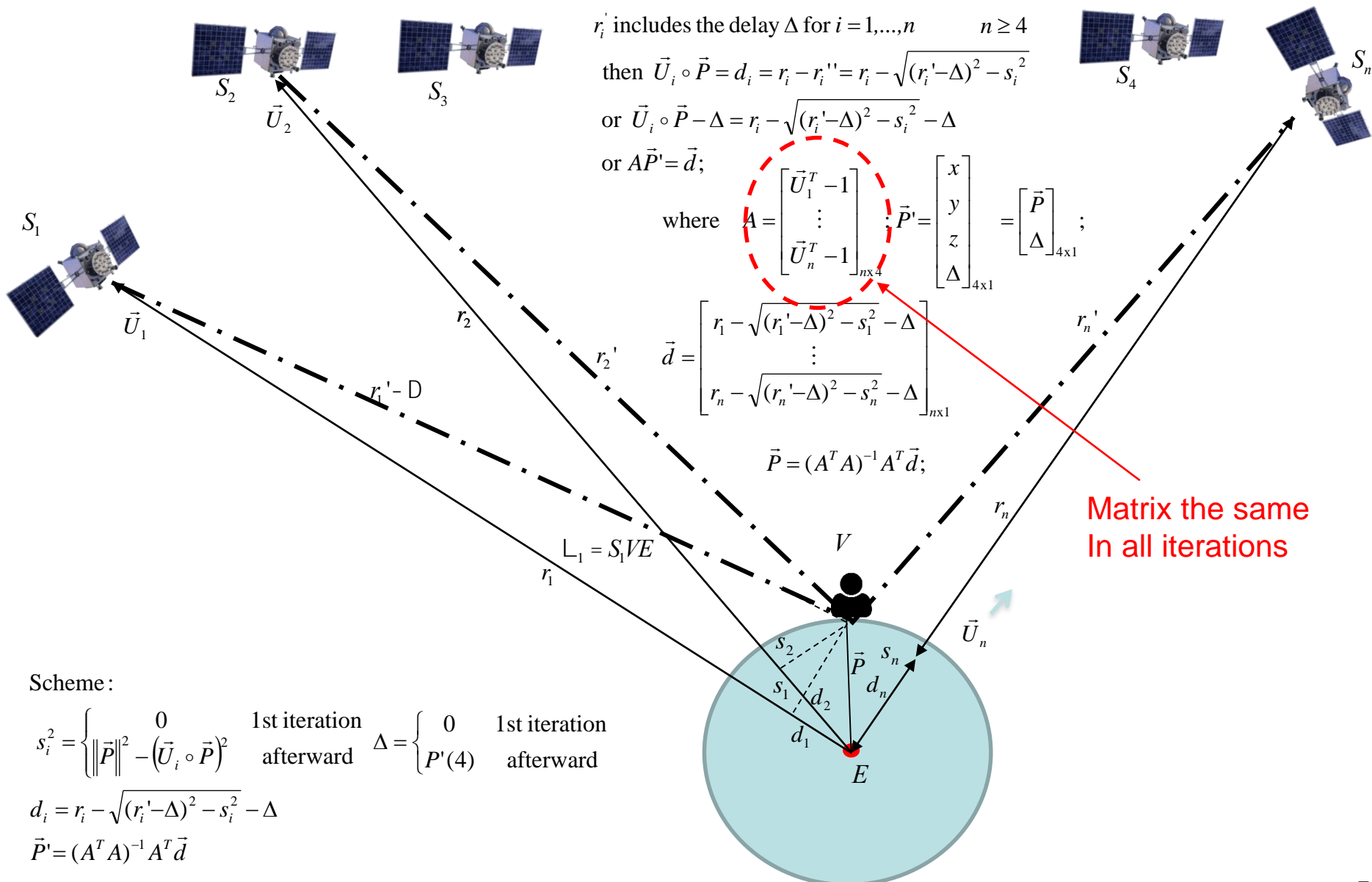
$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} + \begin{pmatrix} \Delta P(1) \\ \Delta P(2) \\ \Delta P(3) \end{pmatrix} \text{ and } T = T + \Delta P(4)$$

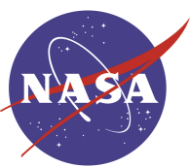


Convergence is based on the Newton's Method (linear regression)



Theory/Analysis: Geometric Trilateration Scheme





Theory/Analysis: Major Error Sources

- Navigation node errors model imperfection knowledge in the transmitting satellite locations and clock offset

$$d'_i = \sqrt{(x_i + v_{xi})^2 + (y_i + v_{yi})^2 + (z_i + v_{zi})^2} \quad i = 1, \dots, n$$

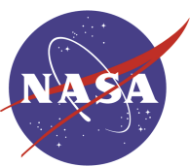
$$v_i = \sqrt{v_{xi}^2 + v_{yi}^2 + v_{zi}^2} \quad i = 1, \dots, n$$

$$v_{xi}, v_{yi}, v_{zi} \sim N(0, \sigma_v^2/3).$$

- Receiver range estimation errors model uncorrected environmental effects such as
 - Transmission medium delays, multipath (systematic bias, not considered in this paper)
 - receiver noise (random error)

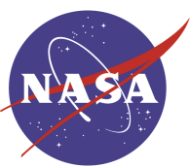
$$r'_i = d'_i + \varepsilon_i \quad i = 1, \dots, n$$

$$\varepsilon_i \sim N(0, \sigma_r^2)$$



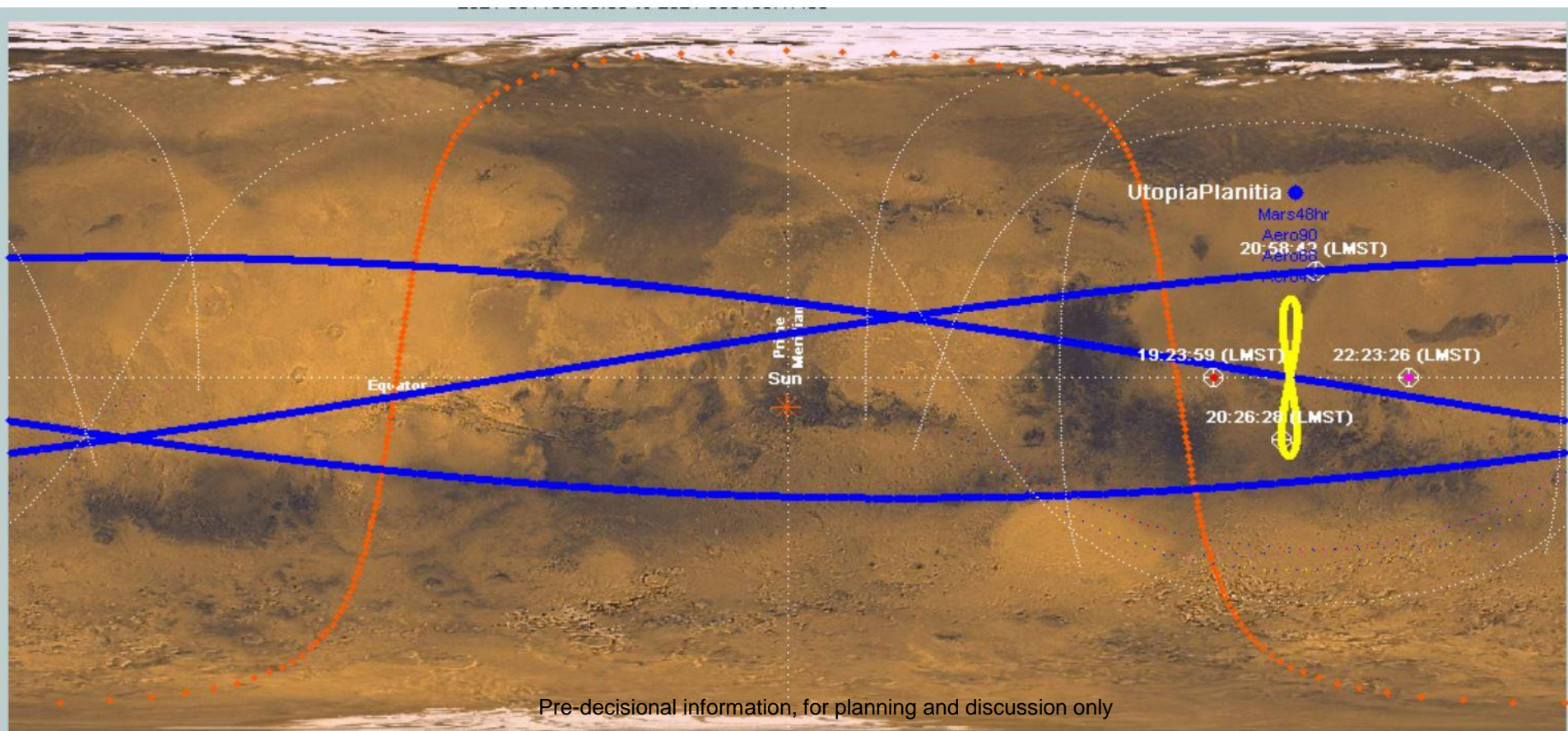
Simulation and Performance: A Notional Mars Navigation Satellite System

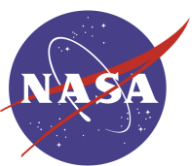
- Examples of human Mars exploration activities that would require positioning support
 - Localizing discoveries and returning to sites
 - Construction and assembling of structures and habitats
 - Entry/descend/landing
 - Approach/rendezvous/docking
 - Mars ascent/orbit insertion
- Traditional deep space tracking methods are Earth-based, and are limited by the speed of light
 - At Mars distance, the one-way-light-time (OWLT) is between 4 and 24 minutes
 - During the final and critical phase of Mars approach when the spacecraft is about to enter the Martian atmosphere, the ground network would not be able to provide timely navigation updates to the spacecraft
- Earth-based tracking cannot cover Mars landing site when it is not in-view of Earth
- Traditional Doppler/ranging is “one-on-one”, and Delta DOR is “two-on-one”
 - Number of orbiting and surface elements is in the order of 10+ or 10's
 - Traditional methods that require pairing one or two ground stations with one spacecraft for a period of time to generate tracking measurements becomes impractical



Simulation and Performance: A Notional Mars Navigation Satellite System

- Aerostationary orbiter 1 (Areo45): 162.5° due East
- Aerostationary orbiter 2 (Areo90): 207.5° due East
- Aerosynchronous orbiter (Areo68): 180° due East, 20° inclined
- Deep Space Habitat (Mars48hr): 180° due East, 149.5° inclined



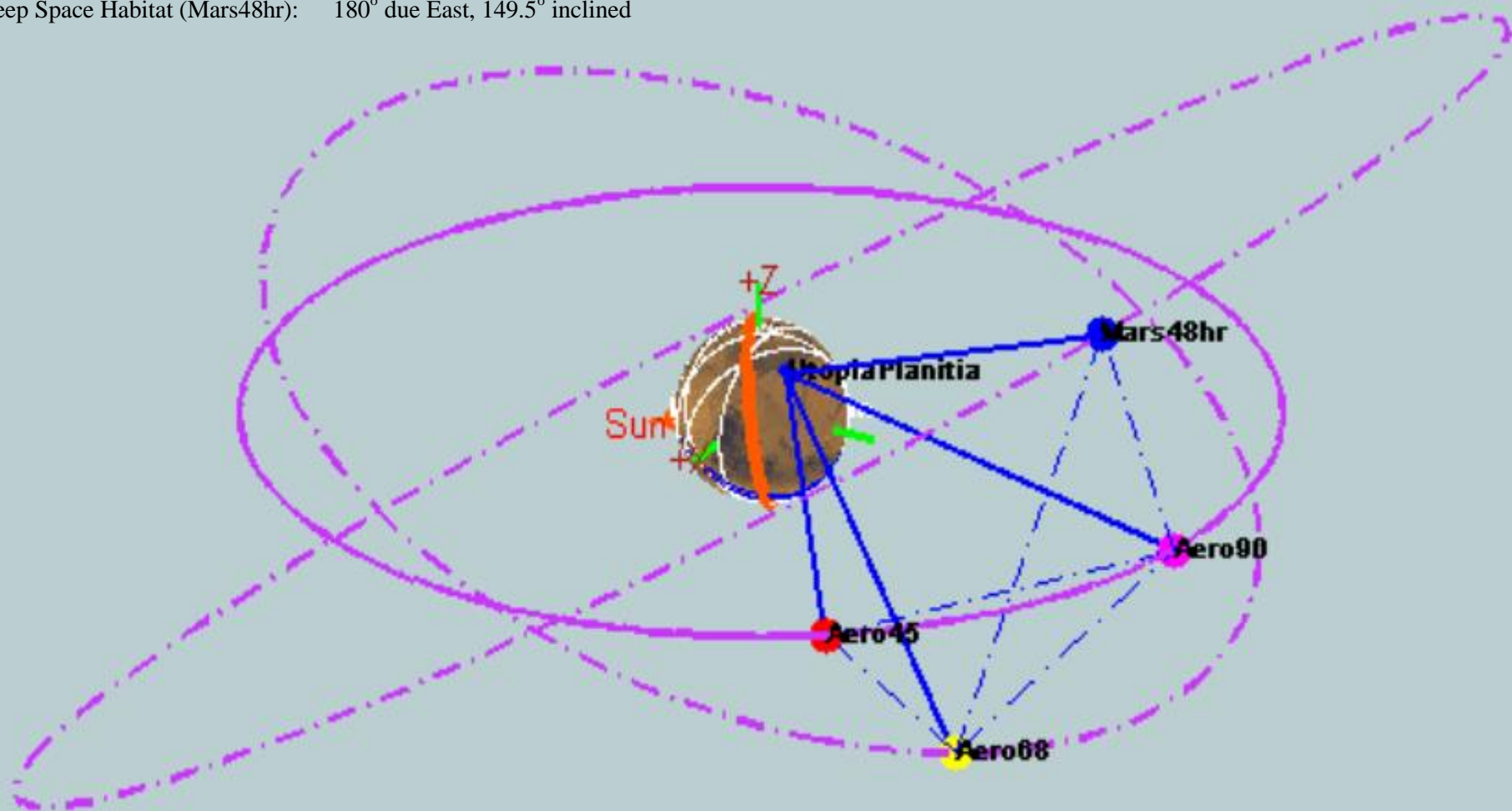


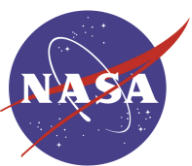
System Concept – Problem Formulation (2 of 3)

Orbits of the Notional Mars Navigation Nodes (3-D View)

Utopia Planitia: 182.5° due East, 46.7° due North
Aerostationary orbiter 1 (Aero45): 162.5° due East
Aerostationary orbiter 2 (Aero90): 207.5° due East
Aerosynchronous orbiter (Aero68): 180° due East and 20° inclined
Deep Space Habitat (Mars48hr): 180° due East, 149.5° inclined

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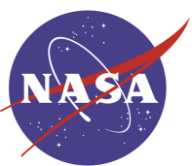
Simulation and Performance: Accuracy Comparison (10,000 runs)

Table 1. σ_{3D} Localization Error standard deviation (cm) of the Traditional NR Scheme. PDOP=113.17.

Traditional NR Algorithm		Navigation Node Error, σ_v							
		0 m	0.5 m	1 m	2 m	5 m	10 m	30 m	35 m
Pseudo-range error, σ_r	0 cm	0.00	3273.85	6547.69	13095.39	32738.48	65476.99	196431.3	229169.9
	0.10 cm	11.27	3273.70	6547.54	13095.23	32738.32	65476.82	196431.1	229169.7
	0.25 cm	28.19	3273.56	6547.35	13095.01	32738.08	65476.58	196430.9	229169.5
	0.50 cm	56.37	3273.51	6547.12	13094.69	32737.71	65476.19	196430.5	229169.1
	1.00 cm	112.74	3274.15	6547.03	13094.24	32737.04	65475.45	196429.7	229168.3
	2.00 cm	225.48	3278.35	6548.30	13094.06	32735.98	65474.10	196428.1	229166.7
	5.00 cm	563.71	3313.95	6563.76	13099.34	32735.15	65471.23	196423.9	229162.4

Table 2. σ_{3D} Localization Error standard deviation (cm) of the New GT Scheme. PDOP=113.17.

Alternate GT Algorithm		Navigation Node Error, σ_v							
		0 m	0.5 m	1 m	2 m	5 m	10 m	30 m	35 m
Pseudo-range error, σ_r	0 cm	0.00	3273.85	6547.69	13095.39	32738.48	65476.99	196431.3	229169.9
	0.10 cm	11.27	3273.70	6547.54	13095.23	32738.32	65476.82	196431.1	229169.7
	0.25 cm	28.19	3273.56	6547.35	13095.01	32738.08	65476.58	196430.9	229169.5
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	5.00 cm	563.71	3313.95	6563.76	13099.34	32735.15	65471.23	196423.9	229162.4



Simulation and Performance: Computation Comparison (10,000 runs)

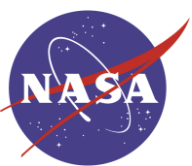
- NR Method takes 6 iterations (RMS) to coverage, whereas the GT scheme takes 36

Table 4. RMS Execution Time (microsec) of the NR Algorithm.

Traditional NR Algorithm		Navigation Node Error, σ_v							
		0 m	0.5 m	1 m	2 m	5 m	10 m	30 m	35 m
Pseudo-range error, σ_r	0 cm	330.18	336.21	329.02	331.08	331.37	331.25	330.67	331.43
	0.10 cm	331.16	329.47	331.22	333.19	331.12	330.13	330.95	334.07
	0.25 cm	330.38	332.01	338.44	331.06	331.34	330.84	331.60	332.16
	0.50 cm	330.48	331.30	334.67	332.06	332.54	332.67	334.23	330.85
	1.00 cm	329.04	331.25	331.45	330.46	332.39	332.33	330.82	332.53
	2.00 cm	331.49	330.84	332.67	334.54	335.20	330.39	337.95	331.21
	5.00 cm	337.32	329.75	331.37	331.63	331.33	330.23	333.06	336.86

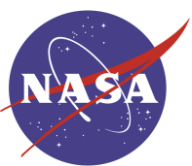
Table 6. RMS Execution Time (microsec) of the GT Algorithm.

Alternate GT Algorithm		Navigation Node Error, σ_v							
		0 m	0.5 m	1 m	2 m	5 m	10 m	30 m	35 m
Pseudo-range error, σ_r	0 cm	1260.52	1299.69	1268.90	1271.10	1269.04	1268.33	1269.61	1264.37
	0.10 cm	1272.40	1298.85	1262.08	1281.08	1271.93	1269.91	1264.77	1266.85
	0.25 cm	1270.05	1268.28	1261.68	1277.05	1270.06	1273.51	1262.43	1278.33
	0.50 cm	1265.67	1266.82	1270.54	1273.19	1266.38	1270.74	1272.08	1276.25
	1.00 cm	1271.30	1261.87	1262.61	1273.36	1270.47	1269.68	1284.95	1270.61
	2.00 cm	1285.71	1262.21	1273.69	1263.89	1266.73	1270.55	1267.84	1275.50
	5.00 cm	1288.90	1266.10	1266.49	1266.28	1276.33	1265.72	1269.76	1271.35



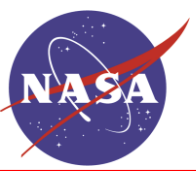
Simulation and Performance: Observations

- The Position Dilution of Precision (PDOP) of the Mars landing site Utopia Planitia with respect to the four orbiter is 113.17
- In the upper left corner of Tables 1 and 2 (NR & GT), the RMS error is zero
- For the left-most column of Tables 1 and 2, in the absence of navigation node error, the localization error \approx PDOP * (pseudo-range error σ_r)
- The localization error is highly dominated by the navigation node error as small as $\sigma_v = 0.5\text{m}$ irrespective of the pseudo-range error σ_r



Concluding Remarks and Future Work

- The NR and the GT schemes give identical accuracy performance
- The NR method converge faster than the GT scheme. As a result, the overall computation time is shorter
- However the computation performance is measured with a random initial guess. For practical operation where the starting value is a recent prior position, the execution speed of the GT scheme is comparable to that of the NR method
- Other advantages of the GT scheme
 - Might allow simpler hardware design due to the need of only one matrix inversion
 - Same algorithm can be used for relative positioning
- Plan forward on Mars navigation satellites
 - Design of navigation signaling scheme that enables fast integer-ambiguity –resolution for carrier phase tracking in the MRNSS environment (poor geometric dilution of precision)
 - Examine the effect of dual and triple frequency receivers to improve the navigation performance at Mars
 - Conduct a hardware-in-the-loop demonstration



Backup
